CE 418.3 - Design in Reinforced Concrete

MIDTERM EXAMINATION

October 28, 2004

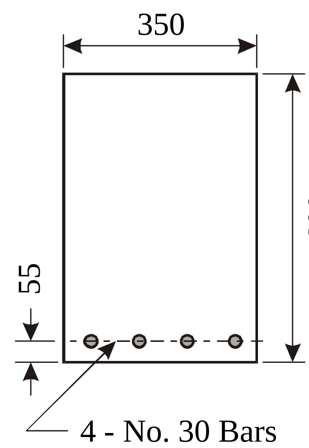
Time Allowed: 2 Hours — Professor: B. Sparling

Notes:

- Closed book examination.
- CPCA Concrete Design Handbook may be used.
- Calculators may be used.
- The value of each question is provided along the left margin.
- Supplemental material is provided at the end of the exam (i.e. formulas).
- Show **all** your work, including all formulas and calculations.
- Clearly specify all assumptions made.

MARKS

QUESTION 1: The reinforced concrete beam shown below was tested in the Structures laboratory under loading conditions that produce a positive moment all along the beam. The beam was constructed with Grade 400 reinforcing steel and concrete with a design strength of $f_c' = 35$ MPa .

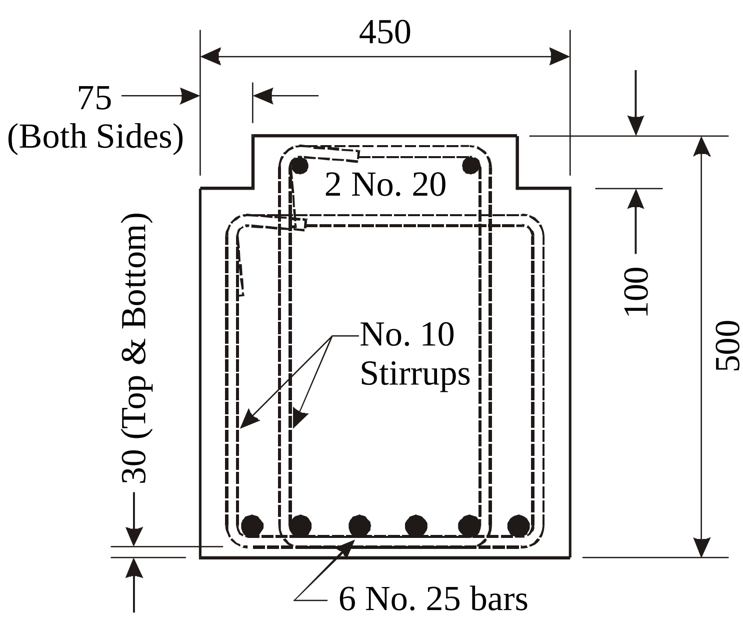


20 **(a)** Estimate the **nominal** (ideal) moment capacity M_n of the beam, based on the principles outlined in Clause 10.1 of CSA-A23.3-94 (including the Whitney stress block described in Clause 10.1.7).

15 **(b)** During the actual test, a strain of $e_s = 0.00075$ was measured in the reinforcing steel when the applied load level was still quite low (i.e. well below the failure load). Assuming that the concrete behaves in a linearly elastic manner ($f_c = e_c E_c$) at this load level, and that it cannot carry any tensile stress, estimate the internal resisting moment in the beam at the location where the strain was measured.

32 **QUESTION 2:** In accordance with the requirements of CSA-A23.3-94, calculate the design ultimate positive bending moment resistance M_r of the reinforced concrete beam shown below. The beam is constructed with Grade 400 reinforcing steel and concrete with a design strength of $f_c' = 30$ MPa .

Hint: Assume that the compressive stress block extends down below the notches at the top of the beam (i.e. a > 100 mm), and check that assumption.



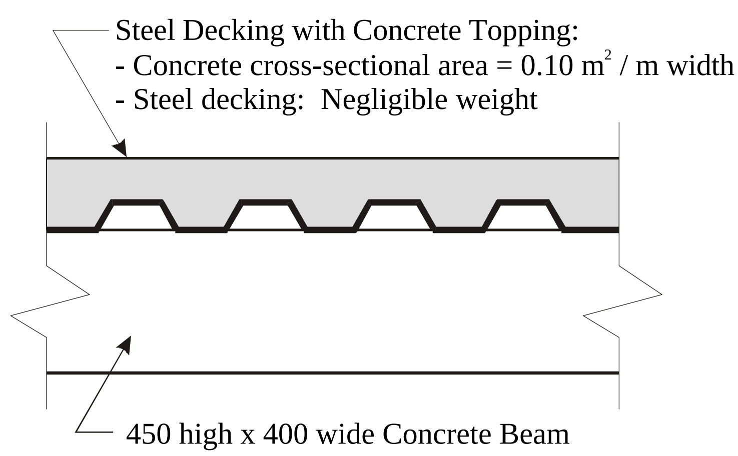
33 **QUESTION 3:** A garage structure features two reinforced concrete frames, each of which consists of three columns poured continuously with a reinforced concrete beam (450 mm high x 400 mm wide) that spans the width of the garage. The roof, which consists of steel decking with a concrete topping, is simply supported by the two concrete frames, spans one-way between the frames, and cantilevers out 1.5 m past the frame at the front of the garage. The roof supports a uniformly distributed specified snow (i.e. live) load of 2.0 kPa.

The concrete frames are constructed with Grade 400 reinforcing steel and concrete with a design strength of $f_c' = 30$ MPa. A clear concrete cover of 40 mm is required over all rebar. The maximum aggregate size is 20 mm.

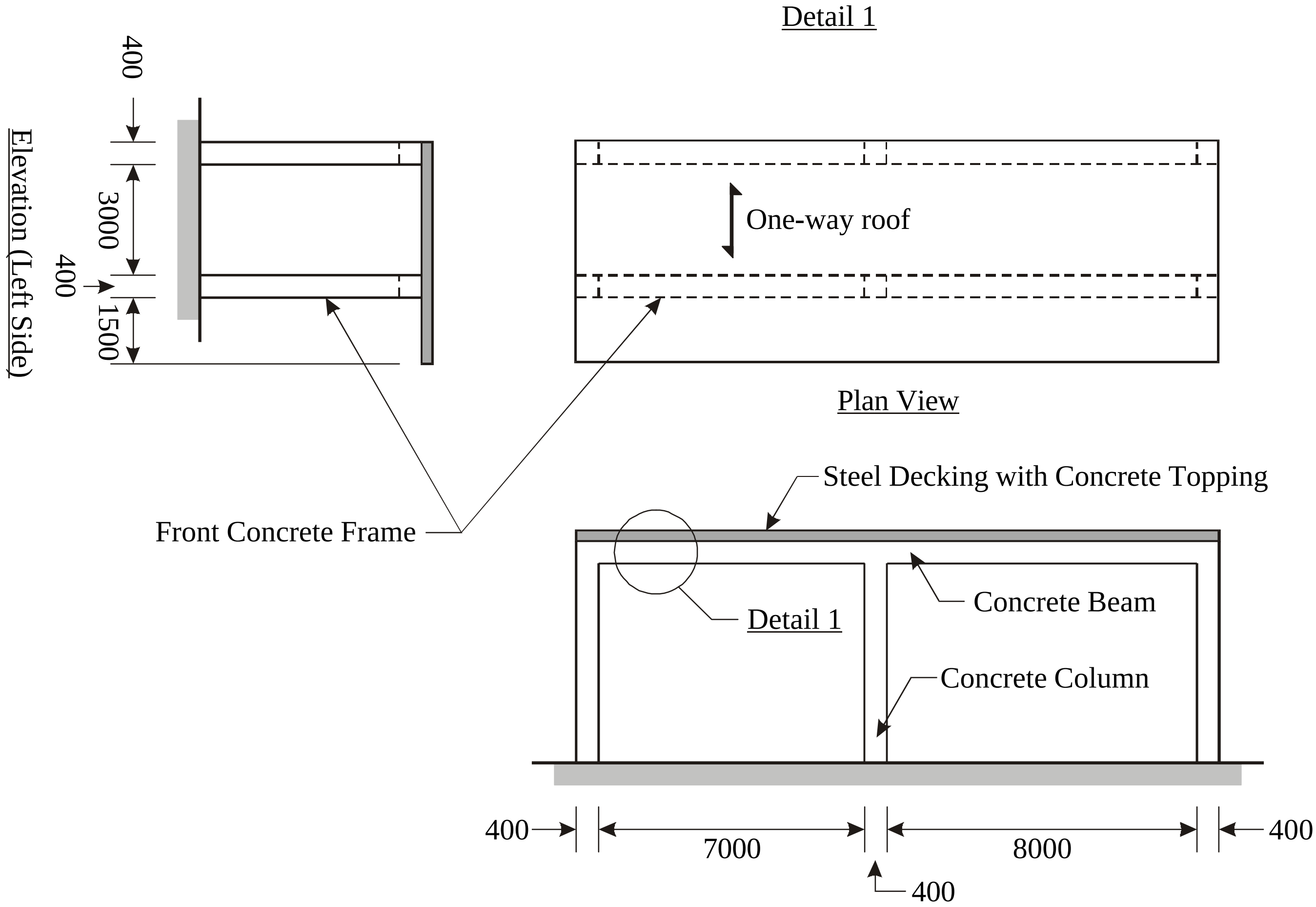
For the frame at the front of the garage (as defined in the sketch below), select the principal reinforcement for the concrete beam at the interior support directly above the central column. The design is to be done in accordance with all relevant requirements of CSA-A23.3-94.

Clearly state and check all assumptions, but **do not** revise your design if the assumptions prove to be incorrect.

Use the appropriate design aids in the Concrete Design Handbook. Include a sketch of the beam showing the selected principal reinforcement.



Detail 1



Elevation (Front)

Supplemental Material:

- **Material Properties:** $\phi_c = 0.6$ $\phi_s = 0.85$ $\alpha_D = 1.25$ $\alpha_L = 1.5$

$$f'_{ct} = \frac{t}{\alpha + \beta t}\; f'_c \qquad \frac{f_c}{f'_c} = 2\left(\frac{\varepsilon_c}{\varepsilon'_c}\right) - \left(\frac{\varepsilon_c}{\varepsilon'_c}\right)^2 \qquad f_{ct} = \frac{2P}{\pi d L} \approx 0.53\sqrt{f'_c}$$

$$E_c = \left(3300\sqrt{f'_c} + 6900\right)\left(\gamma_c / 2300\right)^{1.5} \qquad E_s = 200{,}000 \text{ MPa} \qquad \varepsilon_{cu} = 0.0035$$

$$f_r = 0.6\,\lambda\sqrt{f'_c} \qquad \gamma_c = 2400 \text{ kg/m}^3$$

- **Flexural Analysis:** $\Sigma F_x = 0$ $\Sigma M = 0 \rightarrow M = T\,(j\,d) = C_c\,(j\,d)$

$$C_c = \int_0^c f_c\,\mathrm{d}A \qquad \bar{y}\,C_c = \int_0^c y\,f_c\,\mathrm{d}A \qquad C_c = \left(\phi_c\,\alpha_1\,f'_c\right)(\text{Area}) \qquad T = \phi_s\,A_s\,f_s$$

$$\alpha_1 = 0.85 - 0.0015\,f'_c \geq 0.67 \qquad \beta_1 = 0.97 - 0.0025\,f'_c \geq 0.67 \qquad a = \beta_1\,c$$

$$a = \frac{\phi_s\,A_s\,f_s}{\phi_c\,\alpha_1\,f'_c\,b} \qquad \varepsilon_s = \varepsilon_{cu}\left(\frac{d-c}{c}\right) \qquad \frac{c}{d} \leq \frac{700}{700 + f_y} \qquad \frac{d'}{c} \leq 1 - \frac{f_y}{700}$$

$$(A_s)_{\text{bal}} = \frac{\phi_c\,\alpha_1\,f'_c\,\beta_1\,b\,d}{\phi_s\,f_y}\left(\frac{700}{700 + f_y}\right) \qquad A_{s1} = A'_s\left(\frac{f'_s}{f_s} - \frac{\phi_c\,\alpha_1\,f'_c}{\phi_s\,f_s}\right) \qquad A_{s2} = A_s - A_{s1}$$

$$M_{r1} = \phi_s\,A_{s1}\,f_{s1}\,(d - d') \qquad M_{r2} = \phi_s\,A_{s2}\,f_{s2}\left(d - \frac{a}{2}\right) \qquad \varepsilon'_s = \varepsilon_{cu}\left(\frac{c - d'}{c}\right)$$

- **Flexural Design:** $A_{s_{\min}} = \dfrac{0.2\sqrt{f'_c}}{f_y}\,b_t\,h$ $\rho = \dfrac{A_s}{b\,d}$ $K_r = \dfrac{M_r \times 10^6}{b\,d^2}$

$$\rho_{\text{bal}} = \frac{\phi_c\,\alpha_1\,f'_c\,\beta_1}{\phi_s\,f_y}\left(\frac{700}{700 + f_y}\right) \qquad K_r = \phi_s\,\rho\,f_y\left(1 - \frac{\phi_s\,\rho\,f_y}{2\,\phi_c\,\alpha_1\,f'_c}\right) \qquad M_r \geq M_f$$

$$M_r = \phi_s\,\rho\,f_y\left(1 - \frac{\phi_s\,\rho\,f_y}{2\,\phi_c\,\alpha_1\,f'_c}\right) b\,d^2 \qquad \rho = \frac{\phi_c\,\alpha_1\,f'_c \pm \sqrt{\left(\phi_c\,\alpha_1\,f'\right)^2 - 2\,K_r\,\phi_c\,\alpha_1\,f'}}{\phi_s\,f_y}$$

- **One-Way Floor Systems:** $A_{s_{\min}} = 0.002\,A_g$ $A_{sh} = \dfrac{\left(\phi_c\,\alpha_1\,f'_c\right)\left(h_F\,b\right)}{\phi_s\,f_y}$